

Sec. 11.2A Polynomial Functions and Rational Expressions

Review—Solving Equations: (Remember you can use one variable method, factoring, square root method, completing the square method, or the quadratic formula).

1. $\frac{x^2 - 4}{x + 1} = 3$

$$\begin{aligned} x^2 - 4 &= 3x + 3 \\ x^2 - 3x - 7 &= 0 \\ \frac{3 \pm \sqrt{(-3)^2 - 4(-7)}}{2} &= \frac{3 \pm \sqrt{37}}{2} \end{aligned}$$

2. $(3x - 5) = 4$

$$\begin{aligned} 3x - 5 &= 4 \\ 3x &= 9 \\ x &= 3 \end{aligned}$$

3. $x^2 = 4x$

$$\begin{aligned} x^2 - 4x &= 0 \\ x(x - 4) &= 0 \\ x = 0 & \quad x - 4 = 0 \\ x &= 4 \end{aligned}$$

4. $x^3 - x^2 - 4x + 4 = 0$

$$\begin{aligned} x^2(x - 1) - 4(x - 1) &= 0 \\ (x - 1)(x^2 - 4) &= 0 \\ (x - 1)(x + 2)(x - 2) &= 0 \\ x - 1 = 0 & \quad x + 2 = 0 & \quad x - 2 = 0 \\ x = 1 & \quad x = -2 & \quad x = 2 \end{aligned}$$

5. $x^2 + 5 = 30$

$$\begin{aligned} x^2 &= 25 \\ \sqrt{x^2} &= \sqrt{25} \\ x &= \pm 5 \end{aligned}$$

6. $(x + 3)^2 = 36$

$$\begin{aligned} \sqrt{(x + 3)^2} &= \sqrt{36} \\ x + 3 = 6 & \quad x + 3 = -6 \\ x = 3 & \quad x = -9 \end{aligned}$$

7. $5x^2 + 3x + 1 = 3$

$$\begin{aligned} 5x^2 + 3x - 2 &= 0 \\ (5x - 2)(x + 1) &= 0 \\ 5x - 2 = 0 & \quad x + 1 = 0 \\ 5x = 2 & \quad x = -1 \\ x = \frac{2}{5} & \end{aligned}$$

8. $(x + 3)(2x - 5) = 0$

$$\begin{aligned} x + 3 = 0 & \quad 2x - 5 = 0 \\ x = -3 & \quad 2x = 5 \\ x &= \frac{5}{2} \end{aligned}$$

Factoring the Sum and Difference of Two Cubes:

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Ex: Factor: $x^3 - 8$

$$(x - 2)(x^2 + 2x + 4)$$

Reducing a Rational Expression to Lowest Terms

Rational expression – the quotient of two polynomials, need to look for common factors of the numerator and denominator to simplify (reduce to lowest terms), FACTOR FIRST to reduce.

Ex: Simplify the following:

a. $\frac{3x+9}{x^2-9}$

$$\frac{\cancel{3(x+3)}}{\cancel{(x+3)}(x-3)}$$

$$\boxed{\frac{3}{x-3}}$$

b. $\frac{4x^2+8x}{12x+24}$

$$\frac{\cancel{4x(x+2)}}{\cancel{12(x+2)}}$$

$$\boxed{\frac{x}{3}}$$

c. $\frac{x^2+4x+4}{x^2-16}$

$$\frac{(x+2)(x+2)}{(x+4)(x-4)}$$

Multiplying and Dividing Rational Expressions

**Remember, to multiply fractions you multiply across. To divide, multiply by the reciprocal of the denominator!!

Add and Subtract Rational Expressions

**To add or subtract, the denominator must be the same, then add or subtract the top. Remember to distribute the negative sign when subtracting!

Ex: Perform the indicated operation and simplify the result. Leave your answer in factored form.

a. $\frac{4x^2}{x^2-16} * \frac{x-4}{2x}$

$$\frac{4x^2}{(x+4)(x-4)} \cdot \frac{\cancel{x-4}}{2x}$$

$$\frac{4x^2}{2x}$$

$$\boxed{2x}$$

b. $\frac{2x-5}{3x+2} + \frac{x+4}{3x+2}$

$$\boxed{\frac{3x-1}{3x+2}}$$

c. $\frac{12x}{5x+20} \div \frac{4x^2}{4x^2}$

$$\frac{12x}{5x+20} \div \frac{4x^2}{(x+4)(x-4)}$$

$$\frac{3 \cancel{12x}}{5(x+4)} \cdot \frac{(x+4)(x-4)}{\cancel{4x^2}}$$

$$\boxed{\frac{3(x-4)}{5x}}$$

d. $\frac{x-3}{x+2} - \frac{x+4}{x-2}$

$$\frac{x-2}{x-2} \cdot \frac{x-3}{x+2} - \frac{x+4}{x-2} \cdot \frac{x+2}{x+2}$$

$$\frac{x^2-5x+6 - [x^2+6x+8]}{(x-2)(x+2)}$$

$$\frac{x^2-5x+6-x^2-6x-8}{(x-2)(x+2)}$$

$$\boxed{\frac{-11x-2}{(x-2)(x+2)}}$$

Least Common Multiple Method

**for adding or subtracting polynomials

1. Factor completely the polynomial in the denominator of each rational expression.
2. The LCM of the denominators is the product of each of these factors raised to a power equal to the greatest numbers of times that the factor occurs in any denominator.
3. Write each rational expression using the LCM as the common denominator.
4. Add or subtract the rational expressions.

Ex: Find the least common multiple of $4x^3 - 4x^2 + x$ and $2x^3 - x^2$.

$$x(4x^2 - 4x + 1) \quad x^2(2x - 1) \quad \boxed{x^2(2x-1)(2x-1)}$$

$$x(2x-1)(2x-1)$$

Ex: Simplify $\frac{x}{x^2 + 3x + 2} + \frac{2x - 3}{x^2 - 1}$.

$$\frac{x}{(x+2)(x+1)} + \frac{2x-3}{(x+1)(x-1)}$$

$$\frac{x-1}{x-1} \cdot \frac{x}{(x+2)(x+1)} + \frac{2x-3}{(x+1)(x-1)} \cdot \frac{x+2}{x+2}$$

$$\frac{x^2 - x + 2x^2 + x - 6}{(x-1)(x+2)(x+1)}$$

$$\boxed{\frac{3x^2 - 6}{(x-1)(x+2)(x+1)}}$$

$$\frac{3(x^2 - 2)}{(x-1)(x+2)(x+1)}$$

Simplify Mixed Quotients

1. Treat the numerator and denominator of the mixed quotient separately. Follow by simplifying the resulting rational expression.
2. Find the LCM of the denominators of all rational expressions that appear in the mixed quotient. Multiply the numerator and denominator of the mixed quotient by the LCM and simplify the result.

Ex: Simplify $\frac{\frac{x-2}{x+2} + \frac{x-1}{x+1}}{\frac{x}{x+1} - \frac{2x-3}{x}}$.

$$\frac{\frac{x+1}{x+1} \cdot \frac{x-2}{x+2} + \frac{x-1}{x+1} \cdot \frac{x+2}{x+2}}{\frac{x}{x+1} - \frac{2x-3}{x} \cdot \frac{x+1}{x+1}}$$

$$\frac{\frac{x}{x+1} \cdot \frac{x}{x+1} - \frac{2x-3}{x} \cdot \frac{x+1}{x+1}}{\frac{x}{x+1} \cdot \frac{x}{x+1} - \frac{2x-3}{x} \cdot \frac{x+1}{x+1}}$$

$$\frac{\frac{x^2 - x - 2 + x^2 + x - 2}{(x+1)(x+2)}}{\frac{x^2 - [2x^2 - x - 3]}{x(x+1)}}$$

$$\frac{\frac{2x^2 - 4}{(x+1)(x+2)} \cdot x(x+1)(x+2)}{\frac{-x^2 + x + 3}{x(x+1)} \cdot x(x+1)(x+2)}$$

$$\frac{(2x^2 - 4) \cdot x}{(-x^2 + x + 3)(x+2)}$$

$$\frac{2x^3 - 4x}{-x^3 + x^2 + 3x - 2x + 2x + 6}$$

$$\boxed{\frac{2x^3 - 4x}{-x^3 - x^2 + 5x + 6}}$$

HOMEWORK:

pg 1012-1013 #12 - 78 (m/6)

pg. 1020-1021 #9, 13, 20, 21, 24, 33, 43, 45, 49, 59, 69